

A Lorentz-violating SO(3) model: discussing the unitarity, causality and the 't Hooft-Polyakov monopoles

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In this paper, we extend the analysis of the Lorentz-violating Quantum Electrodynamics to the non-Abelian case: an SO(3) Yang-Mills Lagrangian with the addition of the non-Abelian Chern-Simons-type term. We consider the spontaneous symmetry breaking of the model and inspect its spectrum in order to check if unitarity and causality are respected. An analysis of the topological structure is also carried out and we show that a 't Hooft-Polyakov solution for monopoles is still present.

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I. INTRODUCTION

The possibility of violation of Lorentz and CPT symmetries has been vastly investigated over the latest years [1] - [33]. Although they are in the basis of the modern Quantum Field Theory construction and despite the fact these symmetries are respected by the Standard Model for the elementary particles, one speculates if this scenario is only an effective theoretical description of a low-energy regime. In this case, these symmetries could be violated at an energy near the Planck scale.

The recent papers treat mainly the inclusion, in the gauge sector of the traditional Quantum Electrodynamics (QED), of a Chern-Simons like term

$$\Sigma_{CS} = -\frac{1}{4} \int d^4x \epsilon^{\mu\nu\alpha\beta} c_\mu F_{\nu\alpha} A_\beta, \quad (1)$$

in which c_μ is a constant four-vector which selects a special space-time direction. Such a term would originate an optical activity for the vacuum. This optical activity is presented like a possible explanation for the pattern observed in the detection of ultra-high energy cosmic rays ($E > E_{GZK} \sim 4 \cdot 10^{19} \text{ KeV}$) [34]. Besides that, measurements of radio emission of distant galaxies and quasars detected that the polarization vectors of these radiations are not randomly oriented, as we would expect. Another interesting discussion

has been raised from the investigation of the possibility that this term be radiatively generated from the fermionic sector of ordinary QED, when an axial term, $b_\mu \bar{\psi} \gamma^\mu \gamma^5 \psi$, which also violates Lorentz and CPT, is included [12]-[25].

In the paper [26], the quantization consistency of an Abelian theory which incorporates the term (1), with spontaneous symmetry breaking (SSB), has been contemplated. It was also studied its topological structure, with the discussion on vortex configurations affected by the direction of c^μ in the space-time. Interesting peculiarities were observed in the way the degrees of freedom are distributed amongst the physical modes of the theory after spontaneous symmetry breaking. Remarkable properties are also pointed out in the study of vortex configurations. In this work, we consider a non-Abelian version of this Lagrangian with violation of Lorentz and CPT symmetries. Indeed, we treat a Yang-Mills Lagrangian with internal symmetry SO(3), with the inclusion of a non-Abelian Chern-Simons-type term:

$$\Sigma_{CS}^{NA} = -\frac{1}{4} \int d^4x \epsilon^{\mu\nu\alpha\beta} c_\mu \left(G_{\nu\alpha}^a A_\beta^a - \frac{e}{3} f^{abc} A_\nu^a A_\alpha^b A_\beta^c \right), \quad (2)$$

where, like in the Abelian case, c^μ is a constant four-vector and the latin indices refer to the isospin space. In this context, the SSB is discussed and the quantization consistency is analyzed. This study is carried out by the investigation if unitarity and causality are maintained in the gauge field propagators. The discussion is performed at the tree-level, and we focus on the analysis of the residue matrix associated with each pole of the propagator.

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We also analyze the topological structure of the theory. As we know, in a conventional situation, when the symmetry group is non-Abelian and spontaneous symmetry breaking takes place, the field equations yield a solution which corresponds to a magnetic charge (the 't Hooft-Polyakov monopole). Here, we investigate if this solution is still present and how the Lorentz-breaking term manifests itself in this context.

The paper is organized as follows: in the Second Section, we discuss the SSB and the quantization consistency is investigated from the poles of the gauge field propagator; Section 3 is devoted to the analysis of the field equations and their topological structure; and, finally, in the Section 4, we discuss the results and present our General Conclusions.

II. THE NON-ABELIAN LORENTZ-BREAKING GAUGE-HIGGS MODEL

We begin our analysis by considering the action

$$\Sigma = \int d^4x \left\{ -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} + \frac{1}{2}(D_\mu\varphi)^2 - V(\varphi) \right\} + \Sigma_{CS}^{NA}, \quad (3)$$

where

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + ef^{abc}A_\mu^b A_\nu^c, \quad (4)$$

$$(D_\mu\varphi)^a = \partial_\mu\varphi^a + ef^{abc}A_\mu^b\varphi^c \quad (5)$$

and

$$V(\varphi) = \frac{1}{4}\lambda(\varphi^2 - a^2)^2. \quad (6)$$

As already mentioned, A_μ^a and φ^a are components of vectors in the internal space referring to the SO(3) symmetry. The $\vec{\varphi}$ -field has a non-vanishing vacuum expectation value, $|\langle 0|\vec{\varphi}|0\rangle| = a\hat{u}$, and the SO(3) gauge symmetry is spontaneously broken. We choose the vacuum that points in the internal z -direction,

$$\vec{\varphi}_0 = a\hat{z}. \quad (7)$$

Now we can make use of the fact that we have a local symmetry and choose the so-called unitary gauge, in which $\vec{\varphi}$ lies along the isospin z -axis at every point in space-time

$$\vec{\varphi} = (\rho + a)\hat{z}, \quad (8)$$

where $\vec{\rho} = \rho\hat{z}$ is the physical excitation carried by $\vec{\varphi}$. Notice that the Lorentz-violating Chern-Simons term respects the SO(3) local symmetry; so after SSB occurs, we still have the freedom to fix the gauge, as

in any normal gauge theory. It then turns out that the two isospin components of the gauge field that are orthogonal to the chosen vacuum acquire a mass given by $M^2 = e^2 a^2$, in addition to the topological mass induced by the Lorentz-breaking term. We shall make some comments on the remnant U(1)-symmetry along the z -direction latter. The new gauge action is given as below:

$$\begin{aligned} \Sigma_g = & \int d^4x \left\{ -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} \right. \\ & - \frac{1}{4} \int d^4x \epsilon^{\mu\nu\alpha\beta} c_\mu \left(G_{\nu\alpha}^a A_\beta^a - \frac{e}{3} f^{abc} A_\nu^a A_\alpha^b A_\beta^c \right) \\ & \left. + \frac{1}{2}M^2 [(A_\mu^1)^2 + (A_\mu^2)^2] \right\}. \end{aligned} \quad (9)$$

For the sake of identifying the spectrum of excitations, we can concentrate only on the the quadratic part of the action, that is written as follows:

$$\begin{aligned} \Sigma_g^{quad} = & \int d^4x \left(-\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} - \frac{\mu}{4} \epsilon^{\mu\nu\alpha\beta} v_\mu F_{\nu\alpha}^a A_\beta^a \right. \\ & \left. + \frac{1}{2}M^2 [(A_\mu^1)^2 + (A_\mu^2)^2] \right). \end{aligned} \quad (10)$$

In the equation above, the $F_{\mu\nu}^a$ is the usual Abelian field strength tensor (we are not interested in the interaction Lagrangian),

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a, \quad (11)$$

μ is a mass parameter and v^μ is an arbitrary four-vector of unit length which selects a preferred direction in the space-time ($c^\mu = \mu v^\mu$). This quadratic action can be decomposed in the sum of three actions, one for each gauge field component:

$$\Sigma_g^{quad} = \Sigma_g^1 + \Sigma_g^2 + \Sigma_g^3, \quad (12)$$

with

$$\begin{aligned} \Sigma_g^{1,2} = & \int d^4x \left\{ -\frac{1}{4}F_{\mu\nu}^{1,2} F^{\mu\nu}_{1,2} - \frac{\mu}{4} \epsilon^{\mu\nu\alpha\beta} v_\mu F_{\nu\alpha}^{1,2} A_\beta^{1,2} \right. \\ & \left. + \frac{1}{2}M^2 (A_\mu^{1,2})^2 \right\} \end{aligned} \quad (13)$$

and

$$\Sigma_g^3 = \int d^4x \left\{ -\frac{1}{4}F_{\mu\nu}^3 F^{\mu\nu}_3 - \frac{\mu}{4} \epsilon^{\mu\nu\alpha\beta} v_\mu F_{\nu\alpha}^3 A_\beta^3 \right\}. \quad (14)$$

The quadratic actions for the 1 and 2-components are the same as the one analyzed in the paper [26] and the one referring to the 3-component has its unitarity and microcausality investigated in the paper of reference [11]. In the case of the action corresponding to the 3-component, the authors of [3], [8], [11] study the implications on the unitarity and causality of the theory in

the cases where, for small magnitudes, c_μ is time-like and space-like. The analysis shows that the behavior of these gauge field theories depends drastically on the space-time properties of c_μ . For a purely space-like c_μ , one finds a well-behaved Feynman propagator for the gauge field, and unitarity and microcausality are not violated. Contrary, a time-like c_μ spoils unitarity and causality.

The case of the actions for the 1- and 2-isospin components has been worked out in the paper [26]. Like in the massless case, only for pure space-like c_μ both unitarity and causality can be ascertained. We can confirm these conclusions for the space-like c_μ , by the analysis of the pole structure of the propagators. This is performed by the calculation of the eigenvalues of the residue matrix for each pole. The physical modes (particles with positive norm) have positive eigenvalues. The propagator is given as $\langle A_\mu A_\nu \rangle = i(\mathcal{O}^{-1})_{\mu\nu}$, where $\mathcal{O}_{\mu\nu}$ is a differential (local) operator identified in the action when it is written in the form

$$\Sigma^{quad} = \frac{1}{2} \int d^4x A^\mu \mathcal{O}_{\mu\nu} A^\nu. \quad (15)$$

We then have

$$\begin{aligned} \langle A_\mu A_\nu \rangle_3 &= \frac{i}{D_3} \left\{ -k^2 \theta_{\mu\nu} - \left(-\frac{\alpha D_3}{k^2} \right. \right. \\ &\quad \left. \left. - \frac{\mu^2 (v \cdot k)^2}{k^2} \right) \omega_{\mu\nu} - i\mu S_{\mu\nu} - \mu^2 \Lambda_{\mu\nu} \right. \\ &\quad \left. + \frac{\mu^2 (v \cdot k)}{k^2} (\Sigma_{\mu\nu} + \Sigma_{\nu\mu}) \right\} \end{aligned} \quad (16)$$

and

$$\begin{aligned} \langle A_\mu A_\nu \rangle_{1,2} &= \frac{i}{D_{1,2}} \left\{ -(k^2 - M^2) \theta_{\mu\nu} \right. \\ &\quad \left. + \left(\frac{D_{1,2}}{M^2} - \frac{\mu^2 (v \cdot k)^2}{(k^2 - M^2)} \right) \omega_{\mu\nu} \right. \\ &\quad \left. - i\mu S_{\mu\nu} - \frac{\mu^2 k^2}{(k^2 - M^2)} \Lambda_{\mu\nu} \right. \\ &\quad \left. + \frac{\mu^2 (v \cdot k)}{(k^2 - M^2)} (\Sigma_{\mu\nu} + \Sigma_{\nu\mu}) \right\}, \end{aligned} \quad (17)$$

where

$$D_{1,2} = (k^2 - m_1'^2)(k^2 - m_2'^2), \quad (18)$$

$$D_3 = (k^2 - m_1^2)(k^2 - m_2^2), \quad (19)$$

with

$$m_1^2 = \frac{1}{2} \left[\mu^2 + \mu \sqrt{\mu^2 + 4k_3^2} \right], \quad (20)$$

$$m_2^2 = \frac{1}{2} \left[\mu^2 - \mu \sqrt{\mu^2 + 4k_3^2} \right], \quad (21)$$

$$m_1'^2 = \frac{1}{2} \left[2M^2 + \mu^2 + \mu \sqrt{\mu^2 + 4(M^2 + k_3^2)} \right] \quad (22)$$

$$m_2'^2 = \frac{1}{2} \left[2M^2 + \mu^2 - \mu \sqrt{\mu^2 + 4(M^2 + k_3^2)} \right] \quad (23)$$

We should say that, without loss of generality, we have fixed our external space-like vector as given by $v^\mu = (0, 0, 0, 1)$. We also have chosen a representative momentum for $k^2 > 0$, given by $k^\mu = (k^0, 0, 0, k^3)$. In the propagators, we use the transverse and longitudinal spin operators, $\theta_{\mu\nu}$ and $\omega_{\mu\nu}$, in momenta-space,

$$\theta_{\mu\nu} = g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \quad \text{and} \quad \omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}, \quad (24)$$

and the v^μ -dependent operators,

$$S^{\mu\nu} = \varepsilon^{\mu\nu\kappa\lambda} v_\kappa k_\lambda, \quad \Sigma_{\mu\nu} = v_\mu k_\nu, \quad \lambda \equiv \Sigma_\mu^\mu = v_\mu k^\mu \quad (25)$$

and

$$\Lambda_{\mu\nu} = v_\mu v_\nu. \quad (26)$$

We then have the poles $k_0^2 = k_3^2 + m_1^2$, $k_3^2 + m_2^2$ and $k_0^2 = k_3^2 + M^2$, $k_3^2 + m_1'^2$, $k_3^2 + m_2'^2$ for the propagators $\langle A_\mu A_\nu \rangle_3$ and $\langle A_\mu A_\nu \rangle_{1,2}$, respectively.

A sensible question that arises is whether the number of degrees of freedom is maintained after spontaneous symmetry breaking. This is not a trivial point, since Lorentz symmetry is now violated. To answer it properly, we report here, as it may be found in [26], the calculation related to the poles of the propagators $\langle A_\mu A_\nu \rangle_{1,2}$ given in eq. (17).

With $k_0^2 = k_3^2 + m_1'^2$, we have that the residue matrix reads as below

$$R_1 = \frac{1}{\sqrt{\mu^2 + 4(M^2 + k_3^2)}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \tilde{m}_1^2 - (M^2 + k_3^2) & i\mu \tilde{m}_1 & 0 \\ 0 & -i\mu \tilde{m}_1 & \tilde{m}_1^2 - (M^2 + k_3^2) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (27)$$

with $\tilde{m}_1^2 = m_1'^2 + k_3^2$. We calculate its eigenvalues and find only a single non-vanishing eigenvalue:

$$\lambda = \frac{2|\tilde{m}_1|}{\sqrt{\mu^2 + 4(M^2 + k_3^2)}} > 0. \quad (28)$$

The same procedure and the same conclusions hold

$$R_M = \begin{pmatrix} \frac{k_3^2}{M^2} & 0 & 0 & \frac{|k_3|(M^2 + k_3^2)^{1/2}}{M^2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{|k_3|(M^2 + k_3^2)^{1/2}}{M^2} & 0 & 0 & \frac{(M^2 + k_3^2)}{M^2} \end{pmatrix}, \quad (29)$$

and again we have obtained only a non-vanishing eigenvalue: $\lambda = \frac{1}{M^2} (M^2 + 2k_3^2) > 0$.

The analysis of the residue matrix at the poles yields, for each one, only one non-vanishing and positive eigenvalue. So, the number of degrees of freedom is preserved. We are before a very interesting result: in the ordinary mechanism, the fields A_μ^1 and A_μ^2 , initially with one physical massless vector mode, and then two degrees of freedom, acquire, by the Higgs mechanism, a mass M^2 . This pole corresponds to a mode with three degrees of freedom. In contrast, in the presence of our external background, there occurs a breakdown of this degeneracy and the Higgs mechanism is modified. This makes sense, since in this theory the space-time directions are not treated on the same footing. Here, the physical excitations accommodated in the fields A_μ^1 and A_μ^2 go from two massive to three massive scalar modes, as it is clear from the poles of eq. (17), given by $m_1'^2$, $m_2'^2$ and M^2 , and each one having a corresponding residue matrix with a single non-trivial eigenvalue.

III. ON THE EXISTENCE OF 'T HOOFT-POLYAKOV MONOPOLES

In the paper [26], it was shown that in the modified Electrodynamics there is no room for Dirac-like monopoles, since it would imply in a contradiction between two of the modified Maxwell equations. The effects of Chern-Simons term on 't Hooft-Polyakov monopoles have already been investigated in 2+1-dimensions in [35] and [36]. We now wish to verify whether the known solutions of 't Hooft [37] and Polyakov [38] for an SO(3) theory, which exhibit solutions with magnetic charge, are still present whenever the external background is switched on. The model described by the action of equation (3) leads to the

through for the second zero of D_3 ($k_0^2 = k_3^2 + m_2'^2$): there comes out a unique non-vanishing eigenvalue given by $\lambda = \frac{2|\tilde{m}_2|}{\sqrt{\mu^2 + 4(M^2 + k_3^2)}} > 0$.

Finally, we are left with the consideration of the pole $k_0^2 = M^2 + k_3^2$. The residue matrix reads as follows:

field equations

$$(D^2\varphi)^a = -\lambda\varphi^a (|\vec{\varphi}|^2 - a^2) \quad (30)$$

and

$$(D_\mu G^{\mu\alpha})^a = e f^{abc} (D^\alpha\varphi)^b \varphi^c + c_\mu \tilde{G}^{\mu\alpha a}, \quad (31)$$

where $\tilde{G}^{\mu\nu a} = (1/2)\epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a$ is the dual tensor. We are interested in searching for static and stable solutions to these equations. At infinity, the energy density is given by

$$\mathcal{H} = \frac{1}{2} \left\{ (\vec{G}^{0i})^2 + (-\frac{1}{2}\epsilon_{ijk}\vec{G}_{ij})^2 + (D^0\vec{\varphi})^2 + (D^i\vec{\varphi})^2 + V(\vec{\varphi}) \right\}. \quad (32)$$

This is so because the energy-momentum tensor for the gauge field can be cast according to the expression below, after some algebraic relations are used:

$$\begin{aligned} \Theta^{\mu\nu} &= -G^{\mu\alpha a} G_{\alpha\alpha}^\nu + \frac{g^{\mu\nu}}{4} G^{\alpha\beta a} G_{\alpha\beta}^a \\ &\quad - \frac{1}{4} c^\nu \epsilon^{\mu\beta\rho\sigma} \left(G_{\beta\rho}^a A_\sigma^a - \frac{e}{3} f^{abc} A_\beta^a A_\rho^b A_\sigma^c \right) \end{aligned} \quad (33)$$

and, when c_μ is purely space-like, as we are considering here, it does not contribute to the energy. We need a solution with finite energy and, therefore, at spatial infinity the vacuum conditions

$$G^{\mu\nu a} = 0, \quad (D^\mu\varphi)^a = 0 \quad (34)$$

and

$$V(\varphi) = \frac{1}{4} \lambda (|\vec{\varphi}|^2 - a^2)^2 = 0 \quad (35)$$

must be satisfied. So, as $r \rightarrow \infty$, we must have $|\vec{\varphi}| = a$ and $\vec{A}_\mu = \vec{0}$. The simplest non-trivial $\vec{\varphi}$ that satisfies this condition is

$$\vec{\varphi} = a\hat{r}, \quad (36)$$

where \hat{r} is the unit vector in the \vec{r} direction. This expression, that mixes a vector in the space-time and a vector in the isospace, was called by Polyakov a "hedgehog" solution. So, the Higgs vacuum, $(D_\mu\varphi)^a = 0$, is obtained by the conventional solution, with the most general form of A_μ^a given by

$$A_\mu^a = -\frac{1}{a^2 e} f^{abc} \varphi^b \partial_\mu \varphi^c + \frac{1}{a} A_\mu \varphi^a, \quad (37)$$

with A_μ being the projection of \vec{A}_μ along the direction of $\vec{\varphi}$ in the isospin space. The well-known field-strength tensor obtained by using the potential of equation (37) has the form

$$\vec{G}_{\mu\nu} = F_{\mu\nu} \hat{\varphi}, \quad (38)$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{ea^3} f^{abc} \varphi^a \partial_\mu \varphi^b \partial_\nu \varphi^c \quad (39)$$

and $\hat{\varphi} = \vec{\varphi}/a$. So, as in the conventional SO(3) theory, in the Higgs vacuum, the unique non-vanishing component of $\vec{G}_{\mu\nu}$ is the one associated with the remnant U(1) group rotations about $\vec{\varphi}$. Let us see which kind of equations the field-strength tensor satisfies in the Higgs vacuum:

$$\begin{aligned} \partial_\mu F^{\mu\nu} &= \frac{1}{a} \partial_\mu (\varphi^a G^{\mu\nu a}) \\ &= \frac{1}{a} \{ -e f^{abc} G^{\mu\nu a} A_\mu^b \varphi^c + \varphi^a \partial_\mu G^{\mu\nu a} \} \\ &= \frac{1}{a} \{ \varphi^a (D_\mu G^{\mu\nu})^a \} = \frac{1}{a} \{ \varphi^a c_\mu \tilde{G}^{\mu\nu a} \} \\ &= c_\mu \tilde{F}^{\mu\nu}. \end{aligned} \quad (40)$$

In the calculation above, we have used the field equations and the condition $(D_\mu\varphi)^a = 0$. Furthermore, in a similar calculation, we can show that, in the vacuum,

$$\partial_\mu \tilde{F}^{\mu\nu} = 0. \quad (41)$$

These are just the equations for the Lorentz-violating model with U(1) symmetry in the vacuum, as studied in the papers [9] and [26].

It is now important to point out some comments. In the paper [26], it was shown that the modified Maxwell equations for the Abelian Lorentz-violating model do not allow Dirac magnetic monopoles, for, as it is shown there, it would imply a contradiction between two of the modified Maxwell equations. In the vacuum, we see that the equations now obtained are the same and, so, this prohibition is still present. However, the new definition of the field-strength tensor, as given by the equation (39) (and it is due to

the non-Abelian structure of the model), allows for a magnetic current given by

$$K^\mu = \partial_\nu \tilde{F}^{\mu\nu} = -\frac{1}{2e} \epsilon^{\mu\nu\rho\sigma} f^{abc} \partial_\nu \hat{\varphi}^a \partial_\rho \hat{\varphi}^b \partial_\sigma \hat{\varphi}^c, \quad (42)$$

where $\hat{\varphi}^a = \varphi^a/a$. This current is identically conserved: $\partial_\mu K^\mu = 0$. The conserved magnetic charge is

$$\begin{aligned} g &= \int K^0 d^3x \\ &= \frac{1}{2e} \int_{S^2} \epsilon_{ijk} f^{abc} \hat{\varphi}^a \partial^j \hat{\varphi}^b \partial^k \hat{\varphi}^c (d^2 S)^i, \end{aligned} \quad (43)$$

where the divergence theorem has been used and the integral is taken over the sphere S^2 at infinity. This is the boundary of the static field configuration φ . The integral,

$$n = \frac{1}{8\pi} \int_{S^2} \epsilon_{ijk} f^{abc} \hat{\varphi}^a \partial^j \hat{\varphi}^b \partial^k \hat{\varphi}^c (d^2 S)^i, \quad (44)$$

gives the number of times $\hat{\varphi}(\vec{r})$ covers S^2 as \vec{r} runs over S^2 once. This is so because $\vec{\varphi}(\vec{r})$ must be single-valued. So, we have

$$g = \frac{4\pi n}{e}, \quad (45)$$

and we obtain the usual quantization condition. On the other hand, equation (43), by the Gauss law of the Magnetism, can be written as

$$g = \int_{S^2} \vec{B} \cdot d\vec{S}. \quad (46)$$

In this way, we have

$$B^i = \frac{1}{2e} \epsilon_{ijk} f^{abc} \hat{\varphi}^a \partial^j \hat{\varphi}^b \partial^k \hat{\varphi}^c. \quad (47)$$

By using $\hat{\varphi} = \vec{r}/r$, the equation above yields

$$B^i = \frac{1}{e} \frac{r^i}{r^3}, \quad (48)$$

which gives us

$$g = \frac{4\pi}{e}, \quad (49)$$

the minimal value compatible with the quantization condition (45). We see that, at spatial infinity, in the Higgs vacuum, although the gauge potential $A_\mu = 0$, an electromagnetic field is contributed by the Higgs sector, and we get that, asymptotically, the field configurations indicate the presence of a non-trivial magnetic charge. When observed from infinity, there is indeed a radial magnetic field.

We would like to comment on the existence of 't Hooft-Polyakov solutions. First of all, let us observe that, like in the Abelian case, studied in ref. [26], the temporal gauge is not allowed if $\vec{B} \cdot \vec{c} \neq 0$, as can be seen from eq. (31). In the Abelian case, we have been analyzing vortex-like solutions, and we found out that these solutions may actually show up. Here, we are interested in a radial magnetic field and, so, a constant c_μ does not allow such a condition: we cannot adopt the temporal gauge. This is so due to the presence of an electric field along with the magnetic field, as it happens in the Chern-Simons theory. In order to verify the stability of the 't Hooft-Polyakov solutions, we analyze the energy density of the field configuration, given by eq. (32), written as follows:

$$\mathcal{H} = \mathcal{H}_{temp} + \frac{1}{2} \left((\vec{G}^{0i})^2 + (e\vec{A}^0 \times \vec{\varphi})^2 \right), \quad (50)$$

where \mathcal{H}_{temp} is the energy density we would get in the temporal gauge, generally used in dealing with monopole calculations. This part of the energy, given by

$$E_{temp} = \int d^3x \mathcal{H}_{temp}, \quad (51)$$

can have its minimal calculated by numerical methods. There is a limiting case, due to Bogomol'nyi [39], in which a lower bound can be derived analytically, so that

$$E_{temp} \geq a|g|. \quad (52)$$

In view of this, since the other terms that contribute to the energy are positive, we conclude that we have, also in the Lorentz-violating case, with c_μ spacelike, a lower bound in the energy that assures stability of the monopole configuration. The difference here is that the energy is contributed by the electric field too. However, since the temporal gauge is not possible, the solution of the field equations is much more complicated. In an intermediate region, even if the Bogomol'nyi bound is used, the usual ansatz cannot be adopted, for we do not have spatial isotropy anymore. But, as stated above, we stress the appearance of a radial magnetic field observed in the asymptotic region. This supports the conclusion of the presence of a magnetic charge. However, though we do not know the full solution everywhere (as it happens for the monopoles in the situation where there is no breaking of Lorentz symmetry), it would be interesting to attempt at finding out the explicit solution that interpolates the monopole configuration from infinity to the non-asymptotic region. That is not a trivial task and we intend to devote some time to pursue such an investigation.

Another important discussion is the one related to the fact that, in the direction of the vacuum in isospin

space, the field equations obtained are those of a U(1) Lorentz-breaking model with a Chern-Simons-type term. As mentioned before, these equations do not yield Dirac monopoles. Indeed, the presence of an explicit mass term (Proca-like, for instance) or the mass generation by spontaneous symmetry breaking always yield inconsistencies between the field equations and a Dirac-type monopole. This is also the case if the mass is generated by means of a Lorentz-violating term, as the one introduced in (2). However, we see that, even if Lorentz violation occurs, there persists a solution in which 't Hooft-Polyakov monopoles are present. The explanation is that 't Hooft-Polyakov monopoles are characterized by a field configuration set up by the triplet of Higgses. We would like to stress that the magnetic monopole here identified is originated from the topological character of the vacuum through the Higgs field, and it is actually rather different from a Dirac-like monopole.

IV. CONCLUDING COMMENTS

The purpose of this work is the investigation of two aspects concerning gauge theories with Lorentz- and CPT-symmetry violation: first, we analyze the quantization consistency of a Lorentz and CPT violating non-Abelian model for which the group of symmetry is SO(3), contemporarily with the spontaneous breaking of gauge symmetry. Further, we investigate if the topological structure of this theory yields 't Hooft-Polyakov monopoles.

The study of the quantization consistency of the theory was carried out by pursuing the investigation of unitarity and causality as read off from the gauge-field propagators. The discussion is made at the tree-approximation, without going through the canonical quantization procedure for field operators. We concentrate on the analysis of the residue matrices at each pole of the propagators. We note that we can split the gauge-field propagator in three parts, one for each isospin direction. We thus find two kinds of propagators, when spontaneous symmetry breaking takes place: one for the massless field A_μ^3 , which is in the internal direction of the U(1) remnant symmetry, and another for the massive, A_μ^1 and A_μ^2 , fields. The quantization consistency for a theory with the propagator for the massless A_μ^3 -field was analyzed in the paper [3], [8], [11]. In these work, aspects like unitarity and microcausality were considered and the unique possibility of a consistent theory is the one in which the external vector, c_μ , is spacelike. In the case of the fields A_μ^1 and A_μ^2 , the propagator was studied in the reference [26]. The conclusions on the c_μ vector were the same as in [3], [8], [11]: it must be spacelike. So, the

quantization of our non-Abelian theory can be carried out without problems provided that c_μ is spacelike.

An interesting feature of this model is the breaking of degeneracy of the degrees of freedom. The originally vector modes, which in Lorentz-invariant theories accommodate two (in the case of being massless) or three (if massive) degrees of freedom, are now split into two or three different scalar modes, as it is revealed by the eigenvalues of the residue matrix at the poles of the propagators. This is in agreement with our expectation, since we have no longer space-time isotropy.

We now refer to 't Hooft-Polyakov monopoles. The Lorentz-breaking term, with c_μ spacelike, does not interfere in the finiteness of the energy and we have stable solutions in the Higgs vacuum. The solution which yields the monopole is the usual one. An im-

portant remark must be pointed out that clarifies an apparent contradiction: the equations satisfied by the projection of the field-strength tensor in the vacuum direction are the same as the ones studied in [26], for which Dirac monopoles are forbidden. However, this is not the case here. The 't Hooft-Polyakov monopole is characterized by the configuration of the scalar fields. It appears as an asymptotic solution and the field configuration looks like a magnetic monopole when viewed from infinity.

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- [1] R. Jackiw and S. Templeton, *Phys. Rev. D* **23**, 2291 (1981)
 - [2] S.M. Carroll, G. B. Field and R. Jackiw, *Phys. Rev. D* **41**, 1231 (1990)
 - [3] A. A. Andrianov and R. Soldati, *Phys. Rev. D* **51**, 5961 (1995)
 - [4] M. Goldhaber and V. Trimble, *J. Astrophys. Astron.* **17**, 17 (1996)
 - [5] D. Colladay and V. A. Kostelecký, *Phys. Rev. D* **55**, 6760 (1997)
 - [6] S. Coleman and S. L. Glashow, *Phys. Lett. B* **405**, 249 (1997)
 - [7] D. Colladay and V. A. Kostelecký, *Phys. Rev. D* **58**, 116002 (1998)
 - [8] A. A. Andrianov and R. Soldati, *Phys. Lett. B* **435**, 449 (1998)
 - [9] S. Coleman and S. L. Glashow, *Phys. Rev. D* **59**, 116008 (1999)
 - [10] O. Bertolami, D. F. Mota, *Phys. Lett. B* **455**, 96 (1999)
 - [11] C. Adam and F. R. Klinkhamer, *Nucl. Phys. B* **607**, 247 (2001)
 - [12] R. Jackiw and V. A. Kostelecký, *Phys. Rev. Lett.* **82**, 3572 (1999)
 - [13] W. F. Chen, *Phys. Rev. D* **60** 085007 (1999)
 - [14] M. Pérez-Victoria, *Phys. Rev. Lett.* **83**, 2518 (1999)
 - [15] J. M. Chung, *Phys. Rev. D* **60**, 127901 (1999)
 - [16] J. M. Chung, *Phys. Lett. B* **461**, 138 (1999)
 - [17] W. F. Chen and G. Kunstatter, *Phys. Rev. D* **62**, 105029 (2000)
 - [18] G. Bonneau, *Nucl. Phys. B* **593**, 398 (2001)
 - [19] J. M. Chung and B. K. Chung, *Phys. Rev. D* **63**, 105015 (2001)
 - [20] A.P. Baeta Scarpelli, M. Sampaio and M. C. Nemes, *Phys. Rev. D* **63**, 046004 (2001)
 - [21] A.P. Baeta Scarpelli, M. Sampaio, B. Hiller and M. C. Nemes, *Phys. Rev. D* **64**, 046013 (2001)
 - [22] O. A. Battistel and G. Dallabona, *Nucl. Phys. B* **610**, 316 (2001)
 - [23] M. Pérez-Victoria, *JHEP* **0104**, 032 (2001)
 - [24] A. A. Andrianov, P. Giacconi and R. Soldati, *JHEP* **0202**, 030 (2002)
 - [25] D. Ebert, V.Ch. Zhukovsky, A.S. Razumovsky, *Phys. Rev. D* **70**, 025003 (2004)
 - [26] A.P. Baeta Scarpelli, H. Belich, J. L. Boldo, J. A. Helayel-Neto, *Phys. Rev. D* **67**, 085021 (2003)
 - [27] H. Belich, J.L. Boldo, L.P. Colatto, J.A. Helayel-Neto, A.L.M.A. Nogueira *Phys. Rev. D* **68**, 065030 (2003)
 - [28] Wander G. Ney, J.A. Helayel-Neto, Wesley Spalenza, *PoS WC 2004*, 046 (2004), hep-th/0502233
 - [29] Wander G. Ney, J.A. Helayel-Neto, Wesley Spalenza, hep-th/0502037
 - [30] H. Belich, T. Costa-Soares, M.M. Ferreira, Jr., J.A. Helayel-Neto, *Eur.Phys.J.C* **42**, 127 (2005)
 - [31] H. Belich, T. Costa-Soares, M.M. Ferreira, Jr., J.A. Helayel-Neto, *Eur.Phys.J.C* **41**, 421 (2005)
 - [32] V.B. Bezerra, C.N. Ferreira, J.A. Helayel-Neto, *Phys. Rev. D* **71**, 044018 (2005)
 - [33] Ashok Das, J. Gamboa, J. Lopez-Sarrion, F.A. Schaposnik, *Phys. Rev. D* **72**, 107702 (2005)
 - [34] Orfeu Bertolami, hep-ph/0301191; J. W. Moffat, hep-th/0211167
 - [35] Bayram Tekin, Kamran Saririan, Yutaka Hosotani, *Nucl. Phys. B* **539**, 720 (1998)
 - [36] Bayram Tekin, Kamran Saririan, Yutaka Hosotani, hep-th/9808057
 - [37] 't Hooft, *Nucl. Phys.* **79**, 276 (1974)
 - [38] A. M. Polyakov, *JETP Lett.* **20**, 194 (1974)
 - [39] E. B. Bogomol'nyi, *Sov. J. Nucl. Phys.* **24**, 449 (1976)